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Stabilization of a low-density plasma in a simple magnetic mirror by feedback control†

C. N. LASHMORE-DAVIES
UKAEA Research Group, Culham Laboratory, Abingdon, Berkshire, England
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Abstract. In this paper the conditions for stabilizing an electrostatic instability occurring in a simple magnetic mirror using feedback techniques are discussed. The calculation is made in cylindrical geometry using a model similar to that introduced in 1968 by Arsenin and Chuyanov. In the first part of the paper a diffuse plasma is considered and the effect of varying the locations of the sensing and suppressing systems is examined in the following cases: both suppressor and sensor outside the plasma, only the sensor inside the plasma, and finally both sensor and suppressor inside the plasma. The density threshold is improved by factors of 4, 12 and 36 in the three cases. In the second part of the paper a sharp boundary plasma is considered but phase shift and frequency response are included in the feedback terms. The Nyquist method is used to find a frequency response giving improved stability.

1. Introduction

The possibility of stabilizing a high-temperature plasma by means of feedback methods has recently received a good deal of attention. Since feedback techniques operate with the perturbed plasma quantities it was hoped that such methods might prove to be both simpler and cheaper than alternative approaches if indeed they exist.

The electrostatic instabilities occurring in a plasma can be divided into two general types: *dissipative* and *reactive* (Hasegawa 1968). A characteristic feature of these instabilities is that, for the former, the growth rate is less and usually much less than the oscillation frequency, whereas, for the latter case, the growth rate is of the same order as the oscillation frequency and often larger. The dissipative instability is produced by *one* wave (whose energy can have either sign) being driven unstable owing to a net exchange of energy between the oscillation and the medium. The reactive instability results when *two* waves whose energies are opposite in sign become degenerate in their oscillation frequency but there is no net flow of energy between the oscillation and the medium. In this case the exchange of energy can be thought of as between the two modes of oscillation.

The conditions for stabilizing the two types of instability are very different, at least at threshold (Taylor and Lashmore-Davies 1970). Most of the experiments (and theories) on plasma stabilization by feedback have dealt with dissipative instabilities (Parker and Thomassen 1969, Keen and Aldridge 1969, Simonen *et al.* 1969, Chen and Furth 1969, Keen 1970, Furth and Rutherford 1969). An exception to this is provided by the work of Arsenin and Chuyanov (1968) and Chuyanov *et al.* (1969), where the problem of stabilizing a simple magnetic mirror against a flute-type instability has been considered. The feedback technique considered by Arsenin and Chuyanov consisted of sensing and suppressing from surfaces outside the plasma. This could only be expected to influence surface or large-scale ($\sim$ radius of the system) modes. However, it is these modes which are usually the most dangerous.

† This work appears in abbreviated form in *Feedback and Dynamic Control of Plasmas*, 1970, ed T. K. Chu and A. W. Hendel (Princeton: AIP Conference Proceedings).

A general theory of plasma stabilization by feedback has recently been given by Taylor and Lashmore-Davies (1970). However, the theory only applies close to the stability threshold and for small feedback signals. Here, we consider only the specific case of the reactive instability discussed by Arsenin and Chuyanov (1968) but allow for arbitrarily large feedback signals. Arsenin and Chuyanov confined their analysis to surface modes and rather simple feedback signals.

There are three main aims of this paper. The first is a consideration of the plasma body waves as well as the surface waves. A characteristic feature of plasma body waves in a bounded plasma is the occurrence of nodes in the wave amplitude. The presence of these nodes may be expected to have an important effect on attempts to stabilize the plasma using feedback techniques.

The second aim of the paper is to compare the effect of different locations of the sensing and suppressing surfaces. The following three cases have been analysed and compared: the sensing and suppressing surfaces both outside the plasma, the sensing surface inside the plasma, and both sensing and suppressing surfaces inside the plasma.

Finally, in standard control theory the Nyquist (1932) diagram is much used in order to design a frequency response of the suppressor system giving the desired properties. We shall use the same technique to analyse the effect of different frequency responses on stability.

2. The dispersion equation with feedback

We consider a cylindrical plasma of infinite length whose axis coincides with a uniform constant magnetic field. The plasma is nonuniform and extends from the origin to some radius a. We simplify the problem by introducing a fictitious radial gravitational force to simulate the effect of curvature and gradients in the zero-order magnetic field. For electrostatic perturbations, and assuming the perturbed quantities vary as

$$\varphi(\boldsymbol{r}, t) \propto \varphi(r) \exp\{\mathrm{i}(m\theta - \omega t)\}$$

we obtain the equation already given by Arsenin and Chuyanov (1968):

$$\left(1+\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}^2}N(x)\right)\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2}+\left\{\left(1+\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}^2}N(x)\right)\frac{1}{x}+\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}^2}\frac{\mathrm{d}N}{\mathrm{d}x}\right\}\frac{\mathrm{d}\varphi}{\mathrm{d}x}$$
$$-\frac{m^2}{x^2}\left(1+\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}^2}N(x)\right)\varphi+m^2\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}}\frac{\omega^*}{\omega(\omega+m\omega^*)}\frac{1}{x}\frac{\mathrm{d}N}{\mathrm{d}x}\varphi = 0 \tag{1}$$

where x is a normalized length r/a, $N(x)$ gives the density profile and ω^* is the precession frequency of the ions due to the gravitational drift ($\omega^* = g/\Omega_{\mathrm{i}}a$) where the radial gravitational force was taken as $g(r) = gr/a$. The remaining quantities have their usual meaning and ω_{pi} refers to the cylinder axis. The boundary conditions (Arsenin and Chuyanov 1968) at $x = 1$ are

$$\varphi_{\mathrm{p}}(1) = \varphi_{\mathrm{v}}(1) \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\varphi_{\mathrm{v}}^{\mathrm{I}}(1)-\frac{\mathrm{d}}{\mathrm{d}x}\varphi_{\mathrm{p}}(1)-\frac{\omega_{\mathrm{pi}}^2}{\Omega^2}N(1)\frac{\mathrm{d}}{\mathrm{d}x}\varphi_{\mathrm{p}}(1) = m^2\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}}\frac{\omega^*}{\omega(\omega+m\omega^*)}N(1)\,\varphi_{\mathrm{p}}(1) \tag{3}$$

where φ_{p} is the solution of equation (1) in the plasma, $\varphi_{\mathrm{v}}^{\mathrm{I}}$ is the solution from $x = 1$ to $x = b/a$, and $\varphi_{\mathrm{v}}^{\mathrm{II}}$ is the solution for $x > b/a$.

At this point we introduce the effect of feedback into the model. Following Arsenin and Chuyanov (1968) this is done by imposing certain conditions at the surface $r = b$. Arsenin and Chuyanov specified the potential at $r = b$ whereas we find it more convenient (mathematically) to add the effect of feedback by introducing a surface charge at $r = b$. Assuming that the fluctuating potential has been sensed at some other surface, then a surface charge proportional to the signal detected by the sensor is fed back at $r = b$. In this idealized model it is assumed that the mode to be stabilized can be sensed and fed back perfectly, that is, the sensing and suppressing surfaces consist of an infinite number of infinitesimal electrodes in order to follow the spatial variation ($\sim \exp(im\theta)$). In practice, of course, only a few large electrodes are used. (For experimental details see Chuyanov *et al.* 1969.) It is also assumed that the details of how the suppressor surface receives its charge do not affect greatly the fields within the plasma, and hence the stability conditions.

We may now state the feedback conditions as follows. At $r = b$ we introduce a surface charge density

$$\sigma\left(\frac{b}{a}\right) = \frac{\delta}{a}\epsilon_0 \varphi_p(1) \tag{4}$$

where δ is real and represents the amplification of the feedback circuit, and we have assumed the sensor is at $r = a$. The boundary conditions at $x = b/a$ are then

$$\varphi_v^{I}\left(\frac{b}{a}\right) = \varphi_v^{II}\left(\frac{b}{a}\right) \tag{5}$$

$$\frac{d}{dx}\varphi_v^{II}\left(\frac{b}{a}\right) - \frac{d}{dx}\varphi_v^{I}\left(\frac{b}{a}\right) = \delta\varphi_p(1). \tag{6}$$

The final boundary condition is that

$$\varphi_v^{II} \to 0 \quad \text{as} \quad x \to \infty. \tag{7}$$

3. Stabilization conditions

We now calculate the conditions required for stabilization in a number of special cases.

3.1. *Uniform plasma with sharp boundary at $x = 1$*

This case corresponds to $N(x) = 1$. The solutions of equation (1) in the three regions—plasma, vacuum I and vacuum II—are then:

$$\varphi_p = Ax^{|m|} \tag{8}$$

$$\varphi_v^{I} = Bx^{|m|} + Cx^{-|m|} \tag{9}$$

$$\varphi_v^{II} = Dx^{-|m|}. \tag{10}$$

These solutions represent surface waves. Applying the boundary conditions given by equations (2), (3) and (5)–(7) we obtain the dispersion relation

$$2 + \frac{\omega_{pi}^2}{\Omega_i^2} + \frac{m^2}{|m|}\frac{\omega_{pi}^2}{\Omega_i}\frac{\omega^*}{\omega(\omega + m\omega^*)} + \frac{b}{|m|}\frac{\delta}{a}\left(\frac{a}{b}\right)^{|m|} = 0. \tag{11}$$

The conditions for stable oscillations are

$$\frac{\delta}{|m|}\left(\frac{a}{b}\right)^{|m|-1} > \frac{4}{|m|}\frac{\omega_{\mathrm{pi}}^2}{\omega^*\Omega_{\mathrm{i}}} - 2 \tag{12}$$

and

$$\frac{\delta}{|m|}\left(\frac{a}{b}\right)^{|m|-1} < -\left(2+\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}^2}\right) \tag{13}$$

where we have used the condition $\omega^* \ll \Omega_{\mathrm{i}}$. Conditions (12) and (13) are equivalent but not identical to those of Arsenin and Chuyanov (1968) because we consider charge being fed back and not potential. For stability $|\delta|$ must exceed some critical value.

3.2. *Nonuniform plasma with parabolic density profile*

For this case $N(x) = 1-x^2$. If we also consider the low-density case such that

$$\omega_{\mathrm{pi}}^2 \ll \Omega_{\mathrm{i}}^2$$

then equation (1) reduces to Bessel's equation. The plasma solution is then

$$\varphi_{\mathrm{p}} = A\mathrm{J}_m(px) \tag{14}$$

where the Bessel function of the second kind has been discarded since it diverges at the origin and

$$p^2 = -2m^2\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}}\frac{\omega^*}{\omega(\omega+m\omega^*)}. \tag{15}$$

Consider the case without feedback for a moment. For a given m number we have an infinite set of radial modes each of which becomes unstable above a certain threshold of density. It is easy to see (for example, by considering the plasma bounded by a perfect conductor) that the first radial mode has the lowest threshold, the second, the next lowest, and so on. Therefore the stability threshold for a given m number is determined by the first radial mode.

Now consider the charge fed back at $x = b/a$ as before. The vacuum solutions are again given by equations (9) and (10) and, applying the boundary conditions (2), (3), (5)–(7) again, we obtain the dispersion relation:

$$p\frac{\mathrm{J}_0(p)}{\mathrm{J}_1(p)} = -\delta \tag{16}$$

where we have specialized to the $m = 1$ case. Without feedback ($\delta = 0$) the stability threshold is given by

$$\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}^2} < \frac{x_{01}^2}{8}\frac{\omega^*}{\Omega_{\mathrm{i}}} \tag{17}$$

where x_{01} is the first zero of the J_0 Bessel function. The left hand side of equation (16) is plotted in figure 1. With the aid of this diagram we see that for $\delta \gg 1$ the stability threshold is given by

$$\frac{\omega_{\mathrm{pi}}^2}{\Omega_{\mathrm{i}}^2} < \frac{x_{11}^2}{8}\frac{\omega^*}{\Omega_{\mathrm{i}}} \tag{18}$$

where x_{11} is the first zero of J_1. This represents an improvement in density of approximately 2. However, for negative feedback† figure 1 shows that there is an optimum amplification given by

$$\delta \simeq -2$$

when the threshold density is increased by the factor

$$\frac{(n_0)_{\mathrm{TF}}}{(n_0)_{\mathrm{T}}} = \frac{\text{Threshold density with feedback}}{\text{Threshold density without feedback}} = 4. \tag{19}$$

It should be pointed out that the condition given in equation (18) is actually for infinite amplification. However, once $\delta \gg 1$, this value of the threshold is approached very closely and any further increase in amplification gives only a negligible

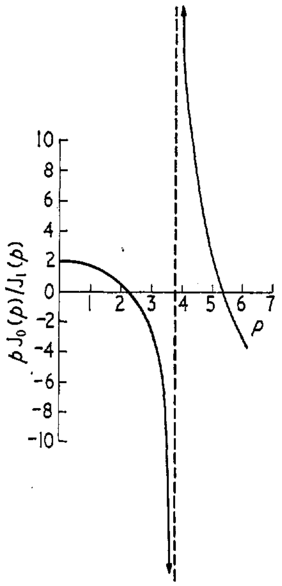

Figure 1. Plot of left hand side of $|m| = 1$ dispersion relation $pJ_0(p)/J_1(p) = -\delta$ when sensor and suppressor are both outside the plasma.

improvement. This is because the effect of the feedback is to reduce the value of the signal that is being sensed, that is, the larger the amplification the smaller the signal to be amplified. However, in this example, increasing the amount of positive feedback increases the stability, if only slightly, whereas increasing the amount of negative feedback above the optimum level decreases stability.

A further improvement in the threshold density can be obtained by sensing at a surface within the plasma (at $x = x_1$ where $x_1 < 1$) but still feeding back outside the plasma at $x = x_2$ ($x_2 > 1$). The dispersion relation then becomes

$$p\frac{J_{m-1}(p)}{J_m(px_1)} = -\delta x_2^{-|m|+1}. \tag{20}$$

Notice that for $|m| = 1$ equation (20) is independent of x_2, that is, for $|m| = 1$ the dispersion properties of the system do not depend on the position of the feedback surface, provided it is outside the plasma.

† In this paper $\delta > 0$ is referred to as positive feedback and $\delta < 0$ as negative feedback.

The function on the left hand side of equation (20) has been plotted in figure 2 for $m = 1$. (Note the dispersion relation is independent of the sign of the mode number m.) With the aid of figure 1 we can find the new threshold density corresponding to the feedback parameter δ. From figure 2 we can see that this time positive

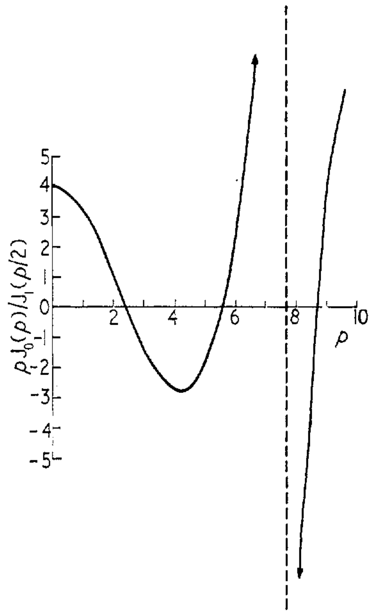

Figure 2. Plot of left hand side of $|m| = 1$ dispersion relation
$$pJ_0(p)/J_1(p/2) = -\delta$$
when suppressor is outside the plasma and sensor is inside the plasma at $x = \frac{1}{2}$.

feedback is more effective than negative feedback (i.e. the larger the value of p at which a root occurs the larger is the threshold density). Furthermore there is an optimum positive value of δ at which the threshold density is a maximum. From figure 2 we can see that this optimum value of δ is approximately 3. The ratio of the threshold densities with and without feedback is now given by

$$\frac{(n_0)_{TF}}{(n_0)_T} = \left(\frac{8\cdot5}{2\cdot4}\right)^2 \simeq 12\cdot5. \tag{21}$$

Thus for a very modest amplification ($\delta \simeq 3$) for positive feedback the density threshold has been increased by an order of magnitude. In the previous example where the suppressor was outside the plasma a similar level of negative feedback produced an increase in the threshold density by a factor of approximately 4.

With such a large increase to the density threshold for the $m = 1$ instability it is interesting to calculate the threshold for the $m = 2$ instability. For $m = 2$ the left hand side of equation (20) is plotted in figure 3. It can be seen that positive feedback is more effective than negative feedback in the sense that approximately the same level of stability is produced for less amplification. For $\delta > 4$ the ratio of the threshold densities with and without feedback is

$$\frac{(n_0)_{TF}}{(n_0)_T} \simeq \left(\frac{10}{2\cdot4}\right)^2 \simeq 17. \tag{22}$$

Thus, for $\delta > 4$, both $m = 1$ and $m = 2$ modes are suppressed up to densities an order of magnitude higher than the value without feedback. (Note: since we are neglecting the frequency response of the suppressor circuit we assume it can respond to all frequencies.)

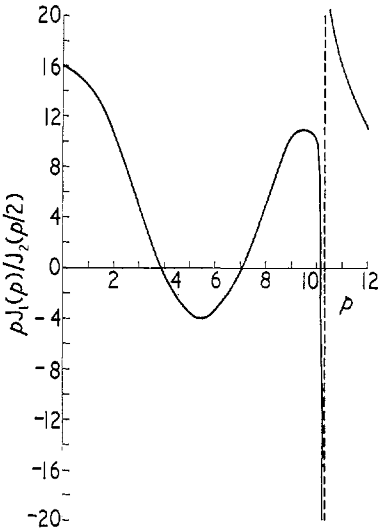

Figure 3. Plot of left hand side of $|m| = 2$ dispersion relation

$$pJ_1(p)/J_2(p/2) = -\delta x_2$$

when suppressor is outside the plasma at $x = x_2$ and sensor is inside at $x = \frac{1}{2}$.

The final example in this section is where both sensing ($x = x_1$) and suppressing surfaces ($x = x_2$) are inside the plasma. Assuming $x_2 > x_1$ we must consider the plasma solutions for the two regions:

$$0 < x < x_2 \rightarrow \varphi_p{}^{\mathrm{I}}$$

$$x_2 \leqslant x < 1 \rightarrow \varphi_p{}^{\mathrm{II}}.$$

$\varphi_p{}^{\mathrm{I}}$ is given by equation (14) and $\varphi_p{}^{\mathrm{II}}$ by

$$\varphi_p{}^{\mathrm{II}} = BJ_m(px) + CY_m(px). \tag{23}$$

For $x > 1$ the vacuum solution is

$$\varphi_v = Dx^{-|m|}. \tag{24}$$

Using the boundary conditions given in equations (2), (3), and (5)–(7) we obtain the following dispersion relation:

$$\frac{J_{m-1}(p)}{J_m(px_1)}\{Y_m(px_2)J_{m-1}(p) - J_m(px_2)Y_{m-1}(p)\}^{-1} = \frac{\pi}{2}x_2\delta. \tag{25}$$

We again obtain the roots of this equation graphically and figure 4 is a plot of the left hand side of equation (25) for $m = 1$. From figure 4 we can see that again both positive and negative feedbacks produce an improvement in the critical density but that positive feedback gives the biggest improvement. For $\delta \geqslant 14$ the improvement

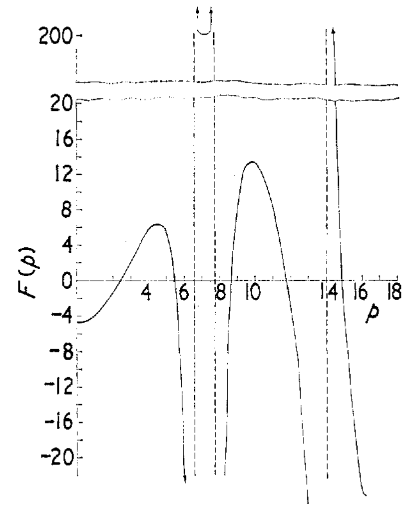

Figure 4. Plot of left hand side of $|m| = 1$ dispersion relation:
$F(p) \equiv \{J_0(p)/J_1(p/2)\}\{Y_1(3p/4)J_0(p) - J_1(3p/4)Y_0(p)\}^{-1} = \frac{3}{8}\pi\delta$
when both sensor and suppressor are inside the plasma at $x = \frac{1}{2}$ and $x = \frac{3}{4}$ respectively.

in the critical density is given by

$$\frac{(n_0)_{\mathrm{TF}}}{(n_0)_{\mathrm{T}}} \simeq \left(\frac{14{\cdot}5}{2{\cdot}4}\right)^2 \simeq 36. \tag{26}$$

Note that if δ is made too large the density threshold is reduced by a factor of 4! In other words δ must be in the range

$$14 \leqslant \delta < 200. \tag{27}$$

4. Frequency dependence of suppressor

So far we have ignored the frequency dependence of the suppressor (i.e. we have assumed constant amplification without phase shift from zero frequency to infinity). The effect of this has been that the modes of oscillation, although stable, remain undamped. In this section we consider the effect of frequency dependence of the suppressor and hence phase shift. For the sake of simplicity we again return to the sharp boundary case of § 3.1. We write the dispersion relation again for this case:

$$2 + \frac{{\omega_{\mathrm{pi}}}^2}{{\Omega_{\mathrm{i}}}^2} + \frac{m^2}{|m|}\frac{{\omega_{\mathrm{pi}}}^2}{\Omega_{\mathrm{i}}}\frac{\omega^*}{\omega(\omega+m\omega^*)} + \left(\frac{a}{b}\right)^{|m|-1}\frac{\delta(\omega)}{|m|} = 0 \tag{28}$$

where δ has now been written explicitly as a function of ω. We now consider three specific cases of complex or frequency-dependent feedback.

4.1. *Feedback proportional to spatial derivative*

For this case we can take

$$\delta = \alpha + i\beta. \tag{29}$$

Substituting this into equation (28) and solving for ω we can see that there is always a root for which

$$\operatorname{Im} \omega > 0 \tag{30}$$

unless

$$\beta = 0. \tag{31}$$

That is, any phase shift other than 0 or π is destabilizing.

4.2. *Feedback proportional to time derivative*

This time we can write for δ

$$\delta(\omega) = \alpha - \mathrm{i}\omega\beta. \tag{32}$$

The dispersion equation for $|m| = 1$ can be written in the form

$$\omega^2 + m\omega^*\omega + \mathrm{i}\frac{\beta}{A}\omega^2(\omega + m\omega^*) + \frac{m^2}{|m|}\frac{\omega_{\mathrm{pi}}{}^2}{\Omega_{\mathrm{i}}}\frac{\omega^*}{A} = 0 \tag{33}$$

where

$$A \equiv 2 + \frac{\omega_{\mathrm{pi}}{}^2}{\Omega_{\mathrm{i}}{}^2} + \alpha.$$

The Nyquist (1932) diagram is often used in plasma stability problems and it is particularly useful in the control problem being considered here. The result of such an analysis on equation (31) shows that there are no conditions corresponding to this form of feedback for which the plasma is stable—the plasma is always unstable. A typical Nyquist diagram for this case is shown in figure 5.

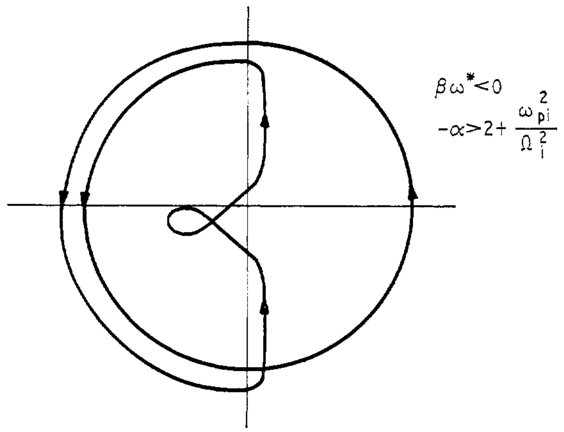

Figure 5. Nyquist plot for frequency response of suppressor proportional to time derivative ($= \alpha - \mathrm{i}\omega\beta$). Contour encircles origin, system unstable.

4.3. *Feedback proportional to the time integral*

For this case we take

$$\delta(\omega) = \alpha + \frac{\mathrm{i}\eta}{\omega}. \tag{34}$$

The dispersion equation can now be written in the form

$$\omega^2 + m\omega^*\omega + \mathrm{i}\frac{\eta}{A}(\omega + m\omega^*) + \frac{m^2}{|m|}\frac{\omega_{\mathrm{pi}}{}^2}{\Omega_{\mathrm{i}}}\frac{\omega^*}{A} = 0. \tag{35}$$

If we let

$$B \equiv \frac{m^2}{|m|}\frac{\omega_{\rm pi}{}^2}{\Omega_{\rm i}}\omega^*$$

then, since we take ω^* positive, B is a positive definite quantity. The Nyquist diagram of equation (34) has six possibilities given by

$$\left.\begin{array}{l} \dfrac{4B}{A} < 0 \\ 0 < \dfrac{4B}{A} < m^2 \\ \dfrac{4B}{A} > m^2. \end{array}\right. \quad \text{and} \quad \begin{array}{l} \eta/\omega^* > 0 \\ \eta/\omega^* < 0 \end{array} \tag{36}$$

Only one of these six possibilities gives stability, namely

$$4B/A < 0 \qquad \eta/\omega^* < 0.$$

The Nyquist diagram for this case is shown in figure 6. Thus, the conditions for

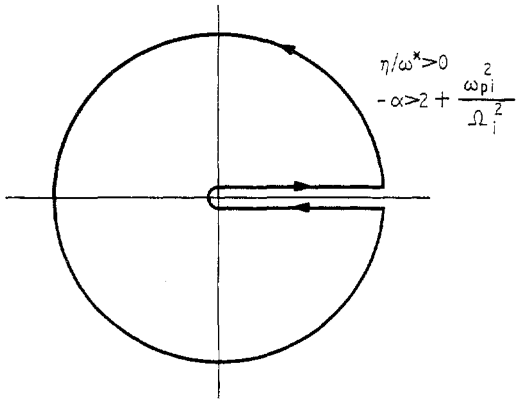

Figure 6. Nyquist plot for frequency response of suppressor proportional to time integral ($=\alpha + i\eta/\omega$). Contour does not encircle origin, system stable.

stabilization with integral feedback are

$$\eta/\omega^* < 0 \tag{37}$$

$$-\alpha >. 2 + \frac{\omega_{\rm pi}{}^2}{\Omega_{\rm i}{}^2}. \tag{38}$$

When the conditions given by equations (37) and (38) are satisfied the plasma is stabilized and the modes of oscillation are damped. Furthermore, the phase shift is no longer critical since by equations (37) and (38) one quarter of the phase plane allows stability.

5. Conclusions

In this paper we have examined the effect of feedback control on a flute-type instability occurring in a low-density plasma. The instability is of the reactive type, that is, it is due to two modes of oscillation, whose energies are of opposite sign, becoming degenerate in their frequency. The conditions for feedback stabilization have been analysed for a number of special cases. In the first set of examples the frequency response of the suppressor system is neglected and amplification at all frequencies without phase shift is assumed. In all cases the electrostatic potential was sensed and charge was fed back.

First of all a surface mode was considered and both positive and negative feedback produced stability for large enough amplification. The resulting modes were purely oscillatory.

Next the effect of feedback on body waves was considered in the following three cases: (i) the sensing and suppressing surfaces were both outside the plasma, (ii) only the suppressing surface was outside, (iii) both surfaces were inside the plasma. Again both positive and negative feedback quenched the instability. However, when the sensing and suppressing surfaces were both outside the plasma, negative feedback was more effective, whereas, in the other two cases, positive feedback appeared to be more efficient.

A characteristic feature of the body-wave case was the existence of an optimum value of the amplification at which the density threshold reached its maximum value. For a further increase in amplification the density threshold for instability either remained almost constant or was actually reduced. This fact appeared to be related to the fact that the effect of the control system is to reduce the signal being sensed. Under these conditions it seems reasonable that there should be an optimum level for the amplification. For the first case the density threshold was increased by a factor of 4 by the control system, in the second case by a factor of 12, and the last by 36.

In the above three cases the results were obtained for the $m = 1$ instability which has the lowest threshold. In the second case it was also verified that the $m = 2$ instability was stabilized under the conditions required to quench the $m = 1$ instability.

In the last part of the paper the frequency response and phase shift of the suppressor system was investigated. For phase shift without frequency dependence all phases (except 0 and π) are destabilizing, even for an initially stable plasma.

A frequency response corresponding to the time derivative is also destabilizing but the time integral response was shown to produce stability under certain conditions. This last case should be useful since it results in damped modes of oscillation and allows stability over one quarter of the phase plane instead of at just two values.

It should be remarked that the type of frequency response which is stabilizing will of course depend on the instability. For some other instability the time derivative might be stabilizing and the time integral response destabilizing.

Finally, although the analysis in this paper refers to the specific example of a flute instability in a magnetic mirror, the results should have a more general relevance since effects such as mode structure, position of sensing and suppressing elements, and the frequency response of the suppressor will clearly be important in other confinement systems.

Acknowledgments

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